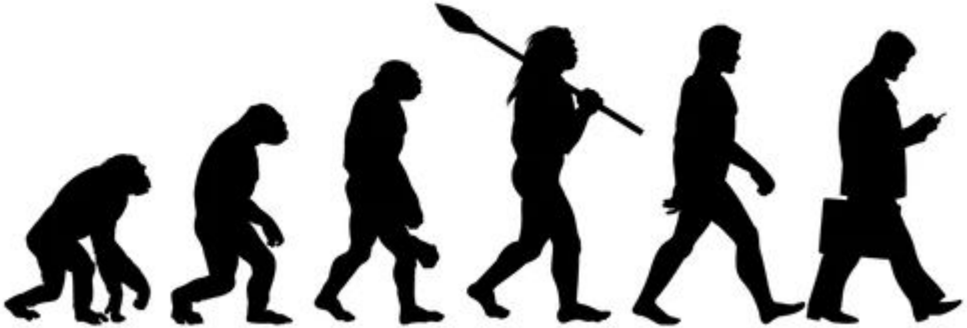
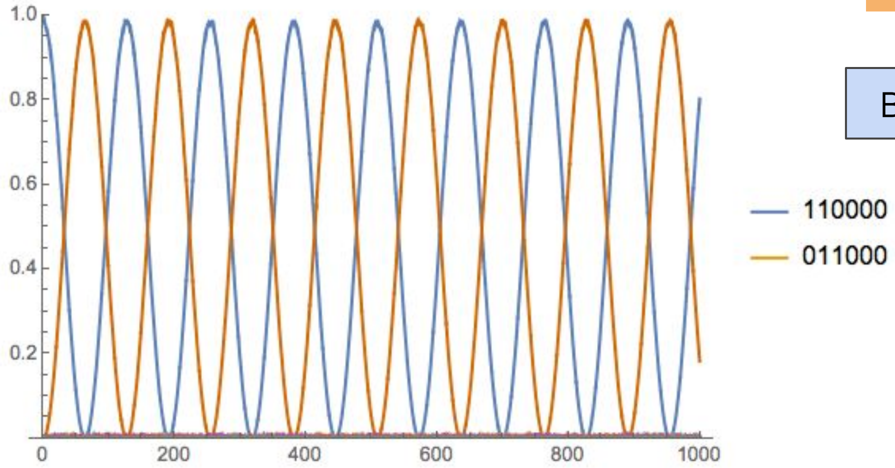
Bounded Pairs in 1-D and 2-D Models

Chemda Wiener, Eliana Feifel, and Lea Santos
Yeshiva University, Stern College for Women

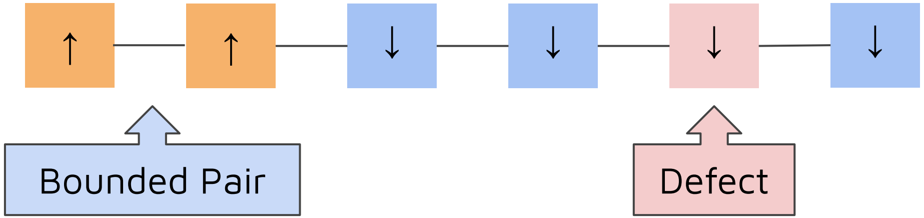
How does our system evolve as a result of its initial state?



Dynamics of a system with initial state |110000>



Bounded pair away from the defect

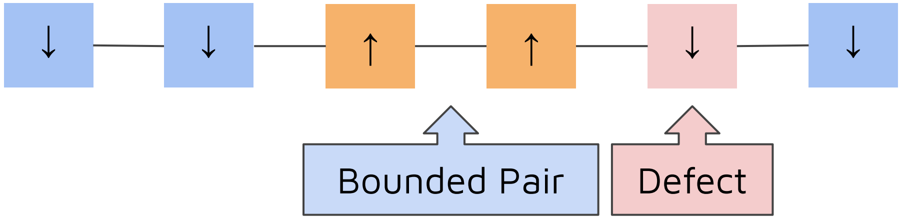


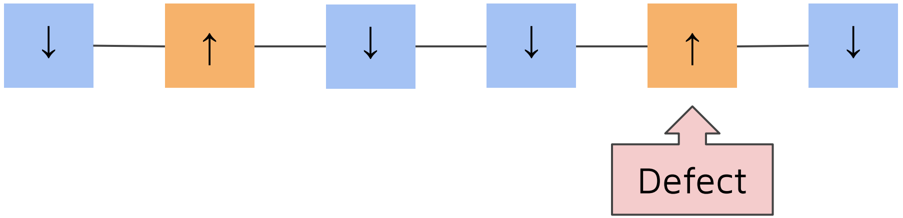
The initial state has an energy of
$J_z/2$ - Eps/2= 0.
This number is from the J_z value: J_z = 10.
The defect does not contribute to the energy of the system as the bounded pair is far from it.

The system can evolve into 2 configurations

Initial state: 001100

J_z = 10, Eps = 10
Energy of system = $J_z/2$ - Eps/2 = 0



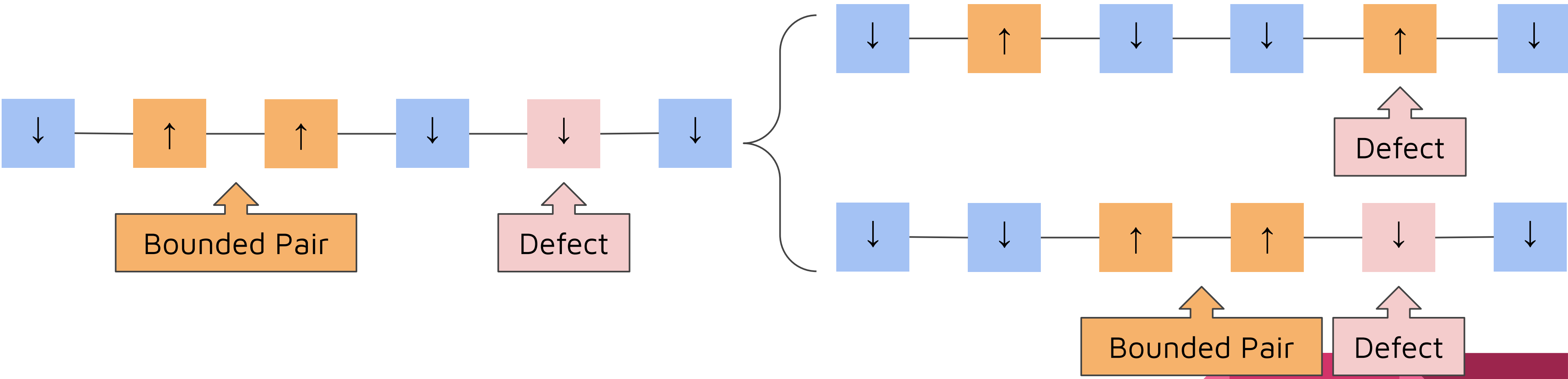


Initial state: 010010

J_z = 10, Eps = 10
Energy of system = $-J_z/2$ + Eps/2 = 0

Quantum Interference

Given the initial state |011000> with the defect still being on site 5, we would expect that this state could have two options to move forward:

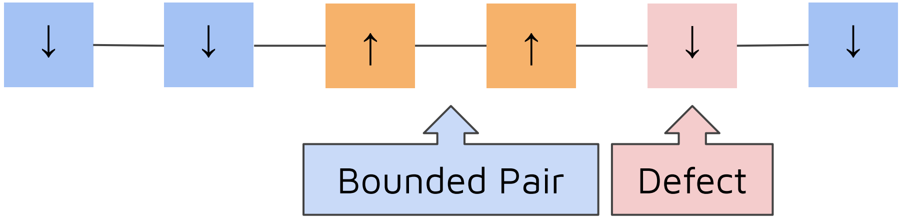


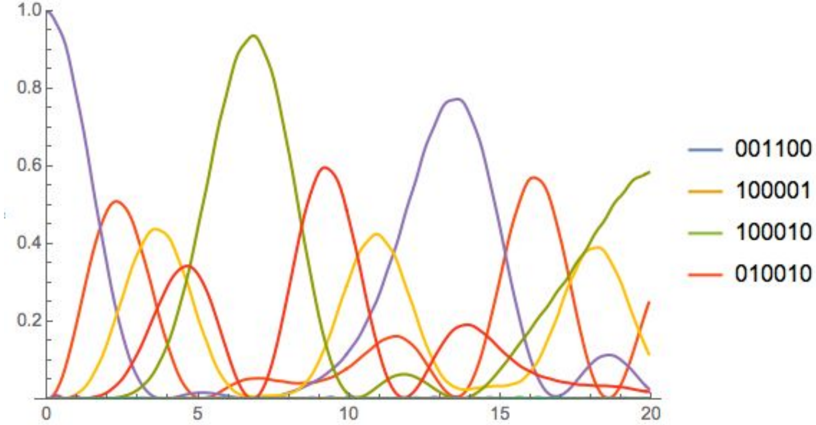
....But neither of these states appear, so we must conclude that there is a destructive interference happening, which we'll refer to as quantum interference.

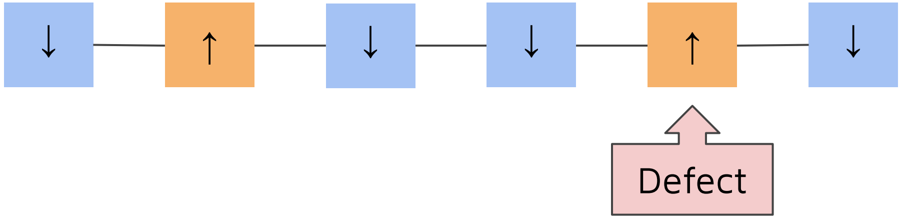
The system can evolve into 2 configurations

Initial state: 001100

J_z = 10, Eps = 10
Energy of system = $J_z/2$ - Eps/2 = 0



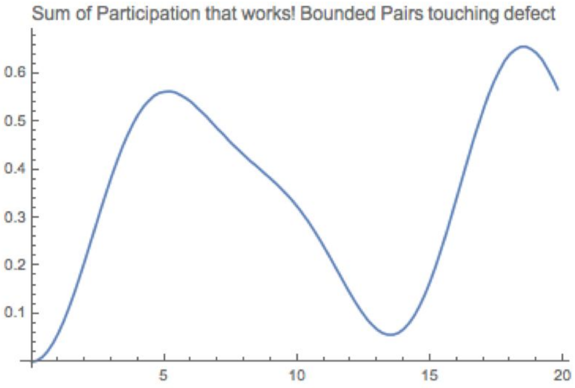


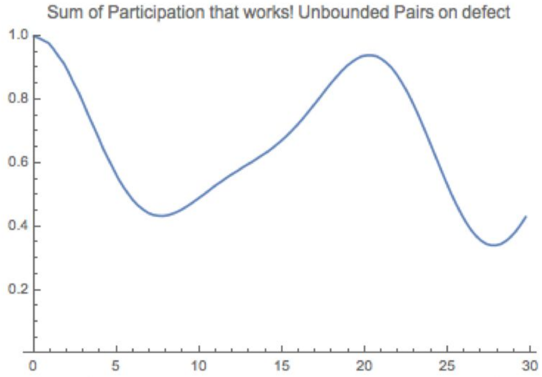


Initial state: 010010

J_z = 10, Eps = 10
Energy of system = $-J_z/2$ + Eps/2 = 0

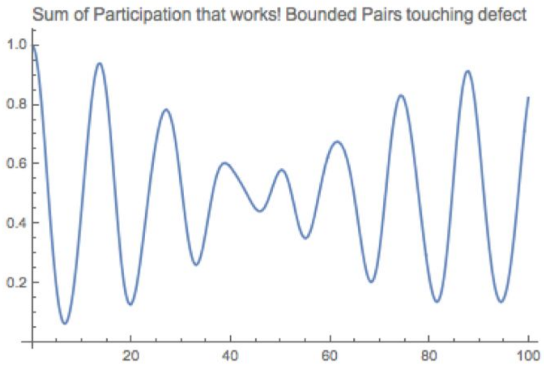
If the defect is on the 5th site: 001100/001010

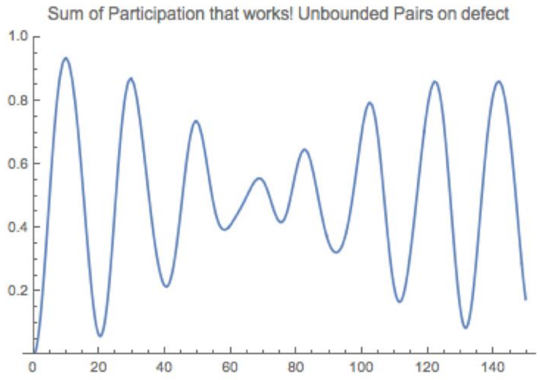




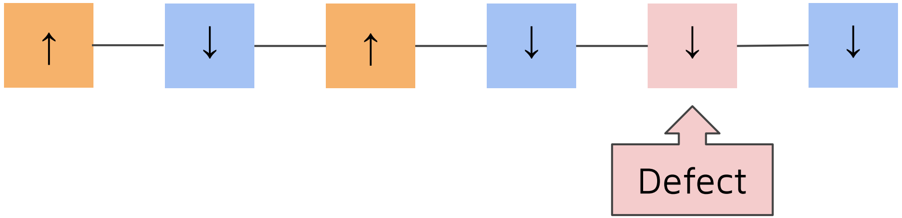
Initial 001100 has the same energy as 001010 and in dynamics that state appears. The sum of theses two graphs equals 1, implying 100% participation.

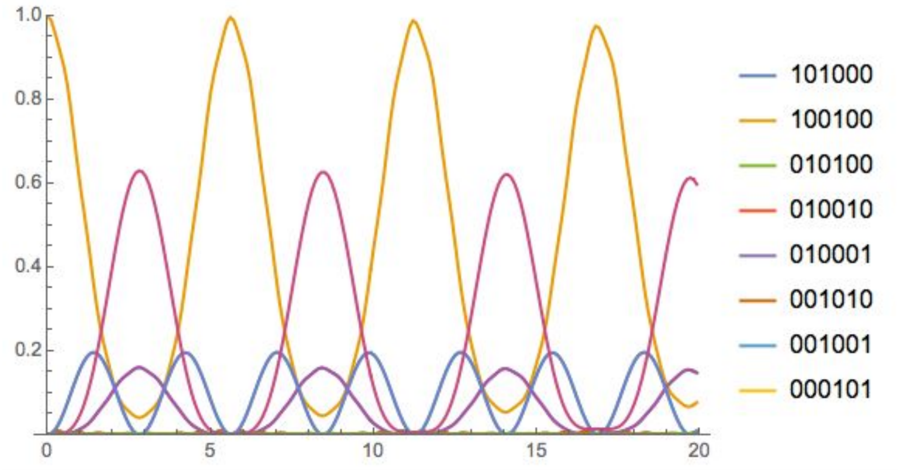
Initial 001010 has the same energy as 001100 and in dynamics that state appears. The sum of theses two graphs equals 1, implying 100% participation.





Dynamics of a system with initial state |101000>

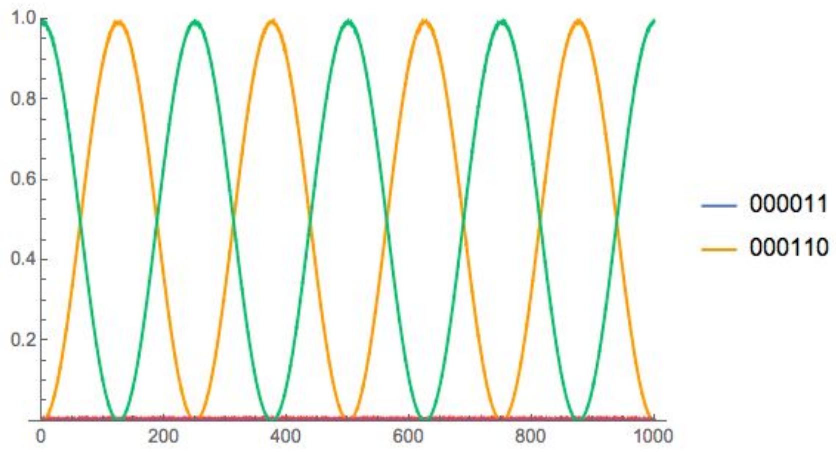




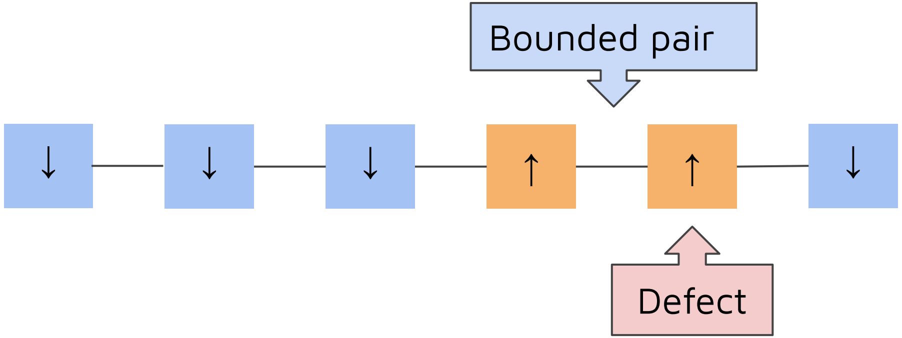
Unbounded up-spins away from the defect

The initial state has an energy of $-J_z/2$ - eps/2 = -10.
This number is from the J_z value: $J_z = 10$.
If we count the Ising interactions, we get the value of $-J_z/2$.
The defect does not contribute to the energy of the system as the up-spins are far from it.

Dynamics of a system with initial state |000110>



Bounded pair on defect



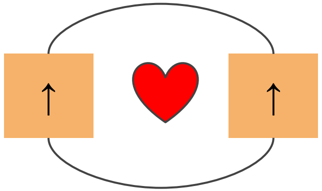
The initial state has an energy of

$J_z/2 + Eps/2 = 5+5 = 10.$

Bound pairs in 1-D are robust

What did we understand up to this point?

Bound pairs cannot split even if we have a defect that would allow for it in energy conservation.



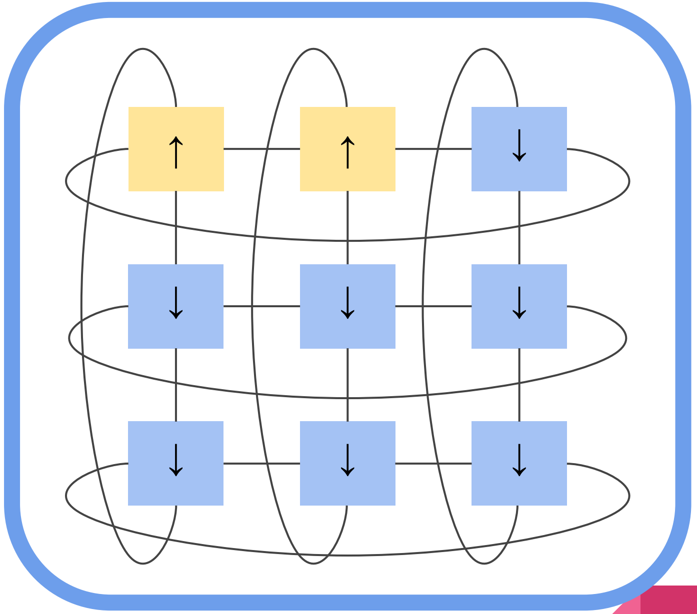
Will this persist in a 2-D system where there are many more possible channels for our excitations to move?

Site Interactions in 2-D

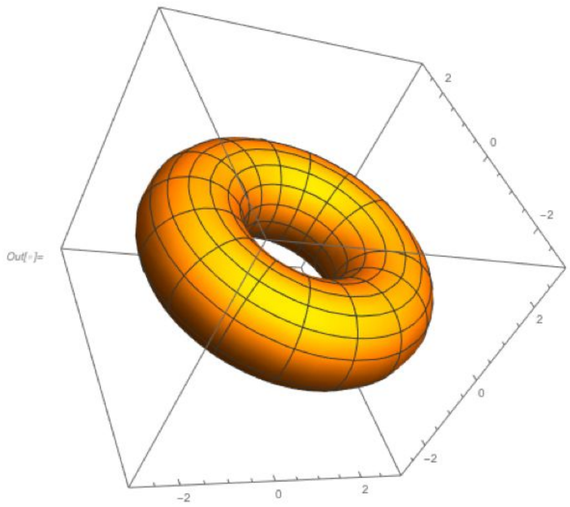
- **The matrices on the right represent a 3x3 2-D system.**

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Our 2-D System



- Similarly to our 1-D system, the first and last site connect, forming a chain. With a 2-D system, the first and last site of every column connect as well. Ultimately, this forms a donut like shape, as displayed in the image to the left.
- It's important to note that the the inner circle does not correspond to a stronger interaction.

Every Basis in the 2-D System

$$\left\{\begin{pmatrix}1&1&0\\0&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&1\\0&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\1&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&1\\0&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&0\\1&0&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}1&0&0\\0&0&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&1&1\\0&0&0\\0&0&0\end{pmatrix},\right.$$

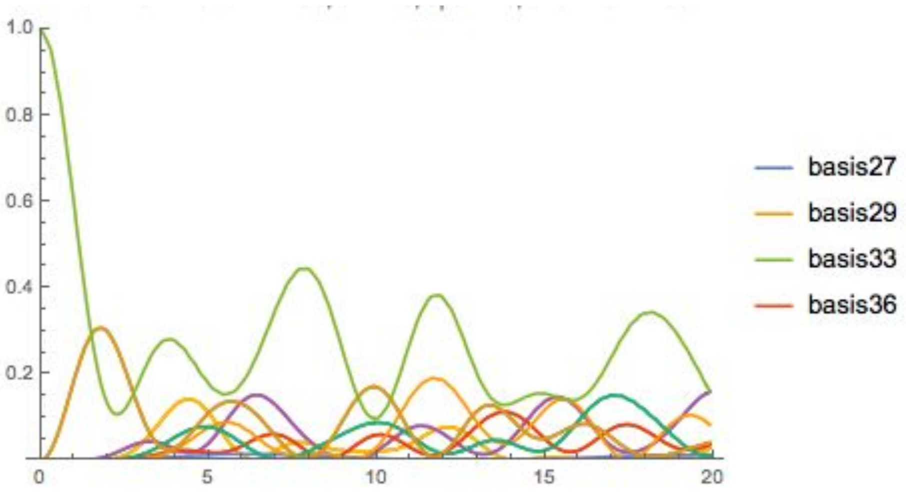
$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&1&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&1&0\\0&0&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&1\\1&0&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&1&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&0&1\\0&0&0\end{pmatrix},$$

$$\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&1\\0&0&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&0\\1&1&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\1&0&1\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\1&0&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\1&0&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&0\\1&0&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&0\\0&1&1\\0&0&0\end{pmatrix},$$

$$\left.\begin{pmatrix}0&0&0\\0&1&0\\1&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&1&0\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&1&0\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&1\\1&0&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&1\\0&0&1\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&0\\1&1&0\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&0\\1&0&1\end{pmatrix}, \begin{pmatrix}0&0&0\\0&0&0\\0&1&1\end{pmatrix}\right\}$$

Stability in the Presence of a Defect?

Defect is on site 1



Bounded pair away from defect

27 $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

29 $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

33 $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

36 $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

Initial state

Different Initial States within the 2-D System

- BP are stable, this becomes more clear as we increase the system size
- And it is really just like a 1-D system in terms of the patterns of its dynamics, even with the defect
- When we introduce a defect to the system, the 2-D system evolves much like the 1-D system

Conclusion

- Working with my dear colleague and friend Eliana
- Benefits of taking the time to study doublons in both 1-D and 2-D models
- Working with the legendary Dr. Lea Santos